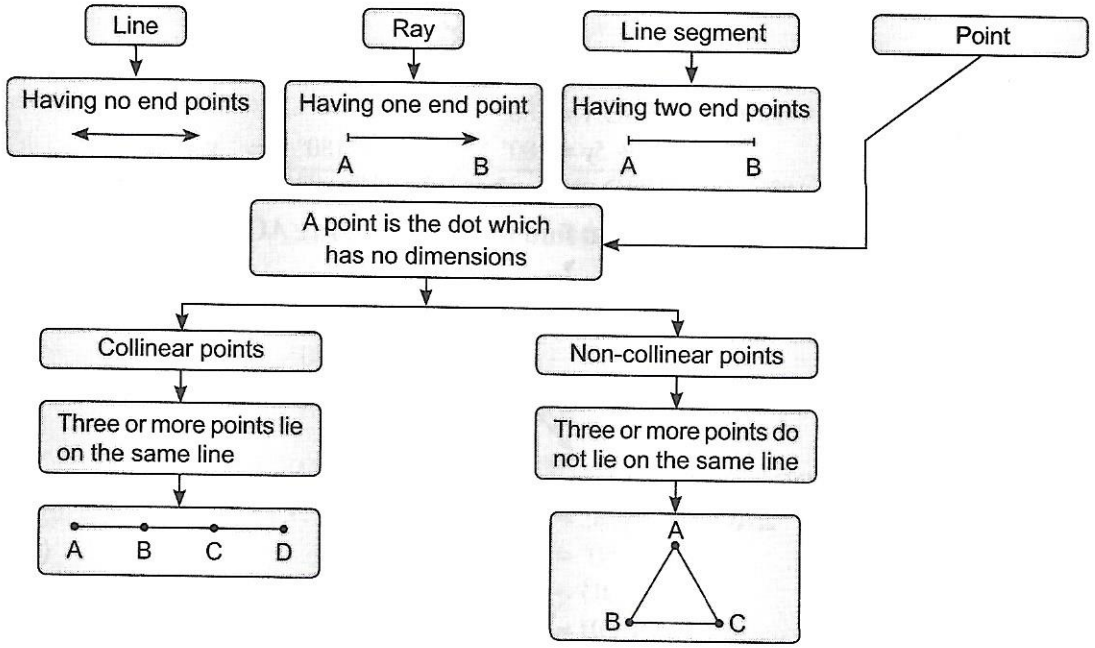
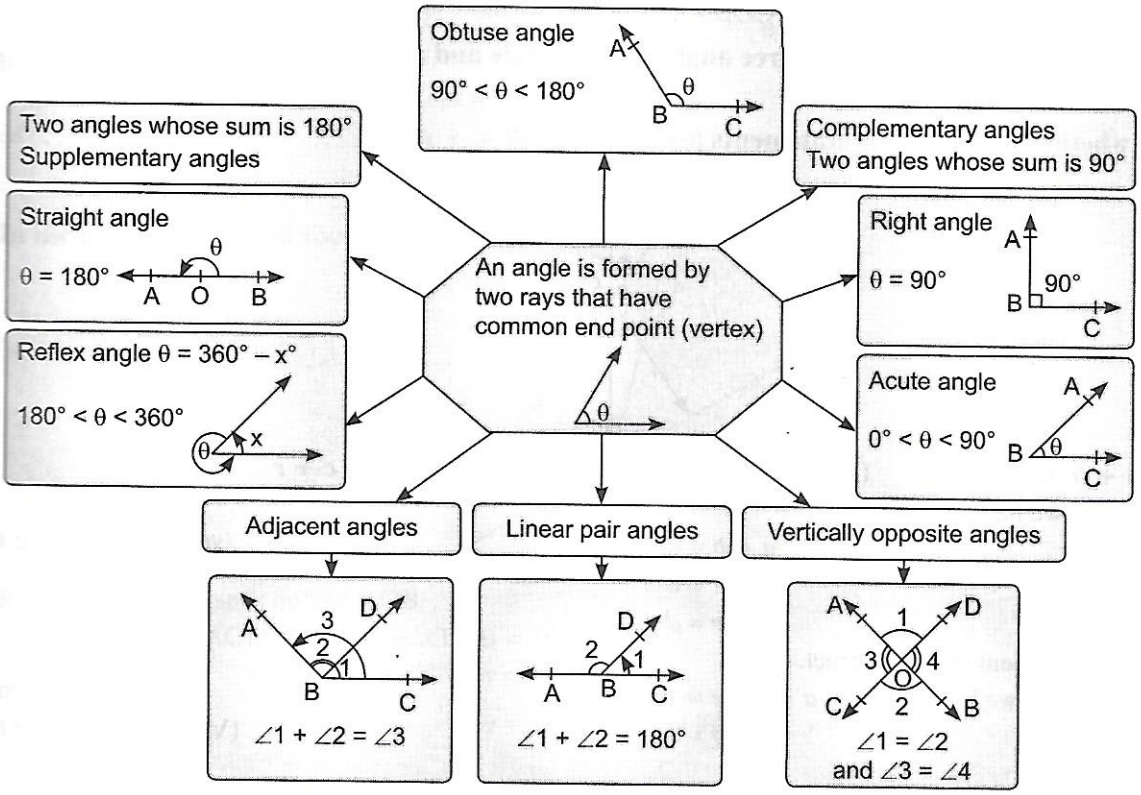
6 LINES AND ANGLES

Basic Terms and Definitions



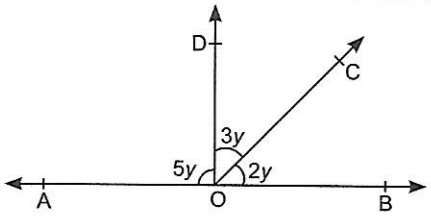
Types of Angles



SOLVED QUESTIONS BASED ON EXERCISE 6.1

Very Short Answer Type Questions [1 Mark]

1. In the given figure, if AOB is a line then find the measure of ∠BOC, ∠COD and ∠DOA. [CBSE 2011]

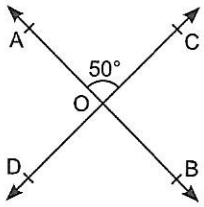


Sol. We have, $\angle BOC + \angle COD + \angle DOA = 180°$

$\Rightarrow \quad 2y + 3y + 5y = 180° \Rightarrow 10y = 180° \Rightarrow y = 18°$

$\therefore \quad \angle BOC = 2y = 2 \times 18° = 36°; \angle COD = 3y = 3 \times 18° = 54°; \angle DOA = 5y = 5 \times 18° = 90°$

2. In the given figure, if ∠AOC = 50° then find the measure of (∠AOD + ∠COB). [CBSE 2010]



Sol. Ray OA stands on line DOC

$\angle AOD + \angle AOC = 180°$ (Linear pair axiom)

$\Rightarrow \quad \angle AOD + 50° = 180°$ (Given: $\angle AOC = 50°$)

$\Rightarrow \quad \angle AOD = 180° - 50° = 130°$

But $\quad \angle COB = \angle AOD$ (Vertically opposite angles)

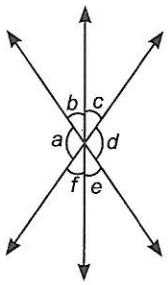
$\Rightarrow \quad \angle COB = 130°$

$\Rightarrow \quad \angle AOD + \angle COB = 130° + 130° = 260°$

3. Write one common fact between three angles of a triangle and a linear pair axiom.

Sol. In both cases, sum of angles is 180°

4. Check whether the following statements are true or not?



(I) $a + b = d + c$ **(II)** $a + c + e = 180°$ **(III)** $b + f = c + e$

Sol. From figure, we have

$a + b = d + e$ (vertically opposite angles)

but $\quad e \neq c$

$\therefore \quad a + b \neq d + c$

$\Rightarrow$ Hence, statement (I) is incorrect.

From the figure, we have $\quad a + f + e = 180°$ (Linear pair axiom) ...(*i*)

But $\quad f = c$ (Vertically opposite angles)

$\Rightarrow$ $a + c + e = 180°$ [From (*i*)]

Hence, statement (II) is true.

Again, $b + c = f + e$ (Vertically opposite angles)

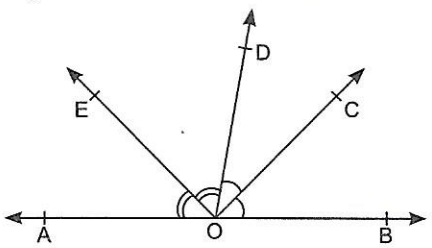
But $c = f$ (Vertically opposite angles)

$b + f = c + e$ (On interchanging c and f)

Hence, statement (III) is also true.

Therefore, statements (II) and (III) are correct.

5. Ray OD stands on line AOB. If ray OC and OE bisects $\angle BOD$ and $\angle AOD$ respectively. Find the $\angle COE$.



Sol. We have $\angle BOC = \angle COD = \frac{1}{2}\angle BOD$ ($\because$ OC is angle bisector of $\angle BOD$)

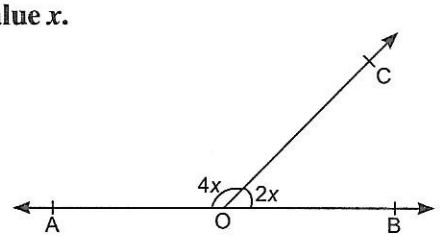
and $\angle AOE = \angle EOD = \frac{1}{2}\angle AOD$ ($\because$ OE is angle bisector of $\angle AOD$)

Now, $\angle AOD + \angle BOD = 180°$ (Linear pair axiom)

$\Rightarrow$ $\frac{1}{2}\angle AOD + \frac{1}{2}\angle BOD = \frac{1}{2} \times 180° = 90°$

$\Rightarrow$ $\angle EOD + \angle COD = 90° \Rightarrow \angle COE = 90°$

6. In the given figure, find the value x.



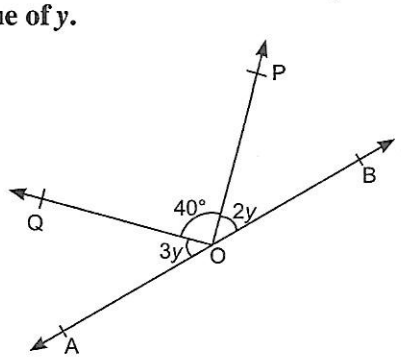
Sol. Ray OC stands on line AOB

$\therefore$ $\angle AOC + \angle BOC = 180°$ (Linear pair axiom)

$\Rightarrow$ $4x + 2x = 180°$

$\Rightarrow$ $6x = 180° \Rightarrow x = \frac{180°}{6} = 30°$

7. In the given figure, find the value of y.



Sol. Ray OP and OQ stands on line AOB

$\therefore$ $\angle AOQ + \angle QOP + \angle POB = 180°$ (Linear pair axiom)

$\Rightarrow$ $3y + 40° + 2y = 180°$

$\Rightarrow$ $5y = 180° - 40° = 140° \Rightarrow y = \frac{140°}{5} = 28° \Rightarrow y = 28°$

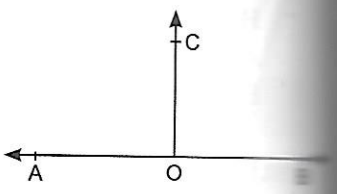
8. If ray OC stands on line AB such that $\angle AOC = \angle BOC$, then show that $\angle BOC = 90°$.

Sol. Ray OC stands on line AOB.

$\therefore \quad \angle AOC + \angle BOC = 180°$ [Linear pair axiom]

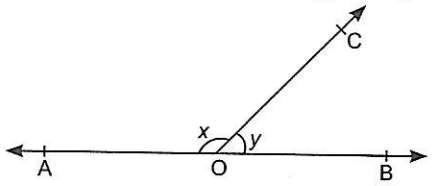
$\Rightarrow \quad 2\angle BOC = 180°$ [$\because \angle BOC = \angle AOC$]

$\Rightarrow \quad \angle BOC = 90°$ Hence Proved.



Short Answer Type Questions I [2 Marks]

9. In the given figure, if x is greater than y by one third of a right angle, find the values of x and y.



Sol. $x = \frac{1}{3} \times 90° + y$ (Given)

$\Rightarrow \quad x = 30° + y$

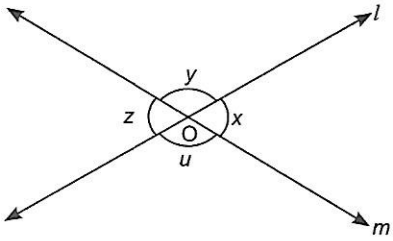
Now, $\angle AOC + \angle BOC = 180°$ (Linear pair axiom)

$\Rightarrow \quad x + y = 180° \Rightarrow 30° + y + y = 180°$

$\Rightarrow \quad 2y = 150° \Rightarrow y = \frac{150°}{2} = 75°$

So, $x = 30° + y = 30° + 75° = 105°$

10. Lines l and m intersect at O. If $x = 45°$, find y, z and u.



Sol. $\angle x$ and $\angle z$ are vertically opposite angles

$\therefore \quad \angle x = \angle z = 45° \Rightarrow \angle z = 45°$

But x and y are linear pair angles

So, $\angle x + \angle y = 180°$ (Linear pair axiom)

$\Rightarrow \quad 45° + \angle y = 180° \Rightarrow \angle y = 180° - 45° = 135°$

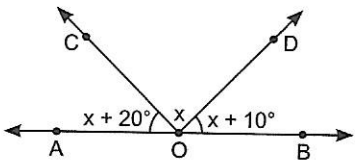
Also, $\angle y$ and $\angle u$ are vertically opposite angles

$\therefore \quad \angle u = \angle y = 135° \Rightarrow \angle u = 135°$

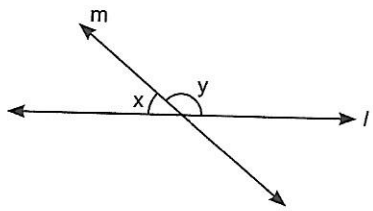
Hence, $\angle z = 45°$, $\angle y = 135°$ and $\angle u = 135°$

PRACTICE QUESTIONS BASED ON EXERCISE 6.1

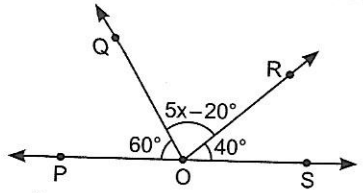
1. The measure of an angle is 8 times its complement. Find the angles.
2. If the measure of an angle is thrice the measure of its supplementary angle, then find the measure of angle.
3. In the given figure, find the value of x.



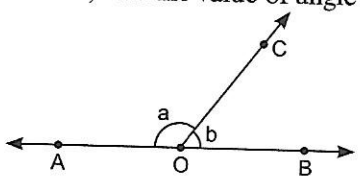
4. In the given figure if $x : y = 2 : 3$, find the value of x and y.



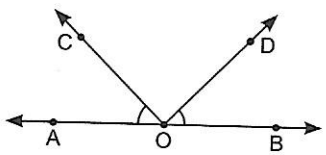
5. In the given figure, POS is a line. Find ∠QOR.



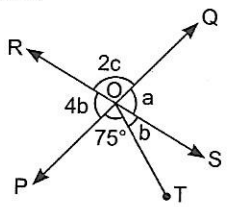
6. In the given figure, ∠AOC and ∠BOC form a linear pair. If ∠b = 75°, find the value of angle a.



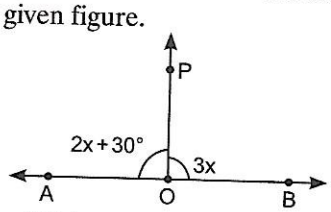
7. In the given figure, if ∠AOC + ∠BOD = 70°, find ∠COD.



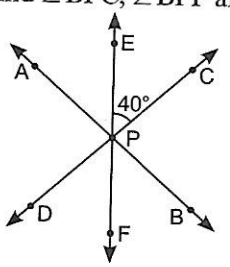
8. In the given figure, two straight lines PQ and RS intersect each other at O. If ∠POT = 75°. Find the value of a, b and c. **[CBSE 2010]**



9. Find the value of x for which AOB becomes a straight line in the given figure.

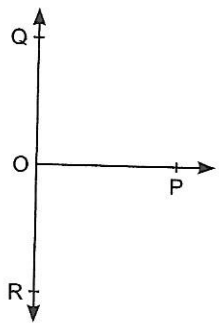


10. If AB, CD and EF are three concurrent lines. Passing through the point P such that PE bisect ∠APC. If ∠CPE = 40°, find ∠BPC, ∠BPF and ∠APF.

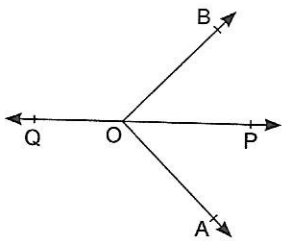


11. Which of the following statements are true and which are false? Give reasons.
   (*i*) Angles forming a linear pair are supplementary.
   (*ii*) If two adjacent angles are equal, then each angle measure 90°.
   (*iii*) If two lines intersect and if one pair of vertically opposite angles is formed by acute angles, then the other pair of vertically opposite angles will be formed by obtuse angles.

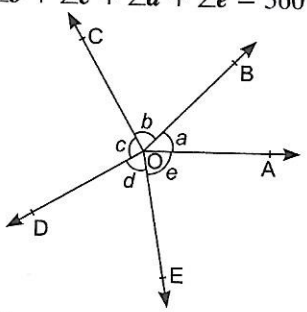
12. In the given figure, ∠POQ and ∠POR are right angles. Show that QOR is a line.



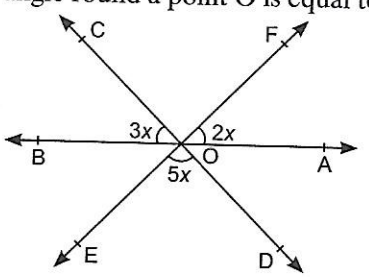
13. Ray OP bisects ∠AOB and OQ is the ray opposite to OP. Show that ∠QOB = ∠QOA.



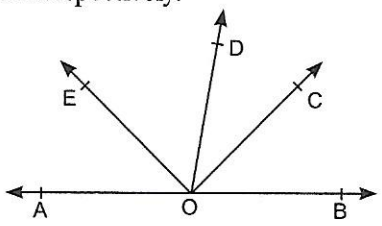
14. Rays OA, OB, OC, OD and OE have the common initial point O as shown in the given figure. Show that ∠a + ∠b + ∠c + ∠d + ∠e = 360°.



15. In the given figure, find the value of x for which sum of all the angle round a point O is equal to 360°.

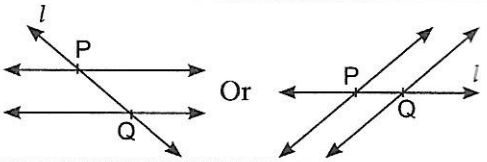
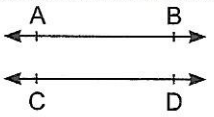
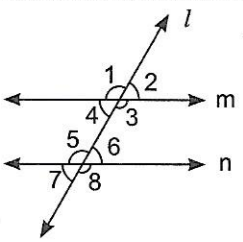
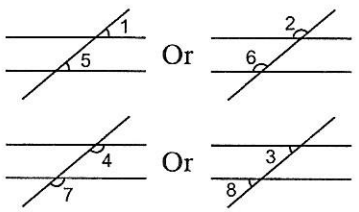
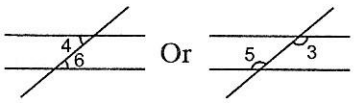
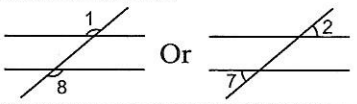


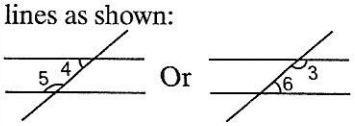
16. In the given figure, ray OD stands on a line AOB, Ray OE and OC are the angle bisector of ∠AOD and ∠DOB respectively.



(*i*) Name the linear pair angles formed. Gi reasons.
(*ii*) Name three pairs of adjacent angles.
(*iii*) Find the measure of ∠EOD and ∠DOC, ∠AOD = x. Prove that ∠EOC = 90°.

Parallel Lines and a Transversal

Transversal (l)	A line which intersects two or more lines at distinct points	
Parallel lines	• Always equidistant from one another • Never intersect	
When a transversal intersects two parallel lines, $m \parallel n$, l is transversal Some pairs of equal (congruent) angles are formed. These are:		
(*i*) Corresponding angles	A pair of angles on the same side of a transversal as shown: 	∠1 = ∠5 ∠2 = ∠6 ∠4 = ∠7 ∠3 = ∠8
(*ii*) Alternate interior angles	A pair of angles on opposite side of transversal but inside the two lines as shown: 	∠4 = ∠6 ∠3 = ∠5
(*iii*) Alternate exterior angles	A pair of angles opposite side of transversal but outside the two lines as shown: 	∠1 = ∠8 ∠2 = ∠7

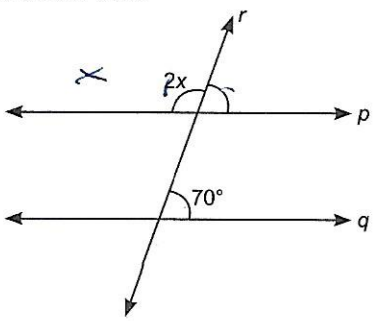
(iv) Co-interior angles	A pair of angles on the same side of transversal but inside the two lines as shown: 	$\angle 4 + \angle 5 = 180°$ $\angle 3 + \angle 6 = 180°$ (Sum of both angles is supplementary)

Note: If a transversal intersects two lines such that, either
- Any pair of corresponding angles are equal, or
- Any one pair of alternate interior angles are equal, or
- Sum of any pair of co-interior angle is supplementary, then these two lines are parallel.

SOLVED QUESTIONS BASED ON EXERCISE 6.2

Very Short Answer Type Questions [1 Mark]

1. In the given figure, $p \parallel q$. Find the value of x. **[CBSE 2010]**



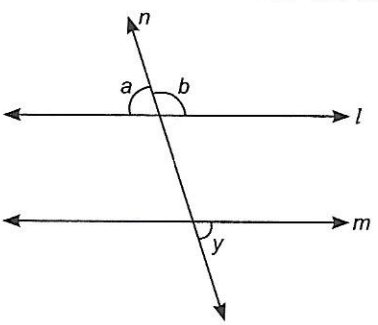
Sol. Since $p \parallel q$ and r is transversal,

$\therefore$ angle which the line r makes with line p is $70°$ (Corresponding angles)

and $2x + 70° = 180°$ (Linear pair axiom)

$\Rightarrow$ $2x = 110° \Rightarrow x = 55°$.

2. In the given figure, if $l \parallel m$ and $\angle a : \angle b = 2 : 3$ then find the value of $\angle y$.



Sol. Given: $\angle a : \angle b = 2 : 3$

$\Rightarrow$ $\frac{a}{b} = \frac{2}{3} = \frac{2k}{3k}$

$\Rightarrow$ $a = 2k$ and $b = 3k$

Now, $\angle a + \angle b = 180°$ (Linear pair axiom)

$\Rightarrow$ $2k + 3k = 180°$

$\Rightarrow$ $5k = 180° \Rightarrow k = 36°$

$\therefore$ $\angle a = 2k = 2 \times 36° = 72°$

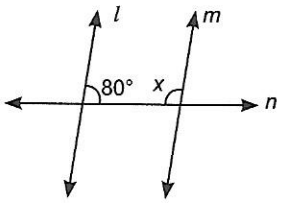
Now, $\angle a$ and $\angle y$ are the alternate exterior angles.

$\therefore$ $\angle a = \angle y$ or $\angle y = \angle a = 72°$

3. If a transversal intersects a pair of lines in such a way that a pair of alternate angles are equal, then what conclusion would you like to draw?

Sol. By alternate interior angles theorem, we conclude that a pair of lines are parallel to each other.

4. If a line $l \parallel m$, n is a transversal in the given figure. Find the value of x.

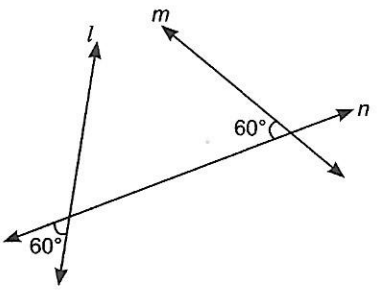


Sol. $l \parallel m$ and n is transversal. Then sum of pair of interior adjacent angles on the same side of transversal is supplementary

$\therefore \quad x + 80° = 180°$

$\Rightarrow \quad x = 100°$

5. Check whether l is parallel to m or not?



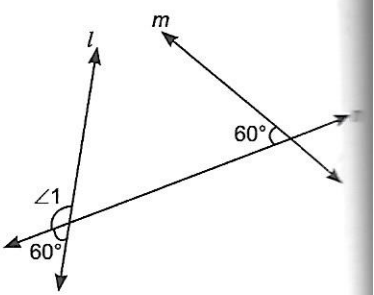
Sol. $\angle 1 + 60° = 180°$ (Linear pair axiom)

$\Rightarrow \quad \angle 1 = 180° - 60°$

$= 120°$

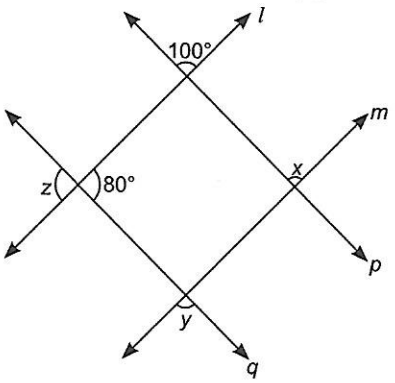
So from the figure, the corresponding angles which makes transversal n with l and m are not equal.

Hence, l is not parallel to m.



Short Answer Type Questions I [2 Marks]

6. Find the value of x and y in the given figure, if $l \parallel m$ and $p \parallel q$.



Sol. As $l \parallel m$ and line p is transversal

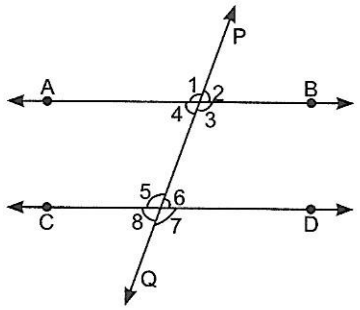
So, $x = 100°$ (Corresponding angles)

Now, $z = 80°$ (Vertically opposite angles)

But $y = 180° - z$ (Corresponding angles)

$\therefore \quad y = 180° - 80° = 100°.$

7. In the given figure, AB || CD, $\angle 2 = 120° + x$ and $\angle 6 = 6x$. Find the measure of $\angle 2$ and $\angle 6$.



Sol. Given AB || CD,

$\Rightarrow$ $\angle 2 = \angle 6$ (corresponding angles)

$\Rightarrow$ $120° + x = 6x$ $(\angle 2 = 120 + x)$

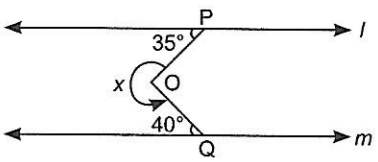
$\Rightarrow$ $120° = 6x - x = 5x$

$\Rightarrow$ $x = \frac{120°}{5} = 24°$

$\therefore$ $\angle 2 = 120° + x = 120° + 24° = 144°$

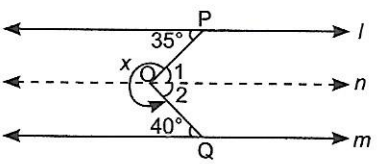
and $\angle 6 = 6x = 6 \times 24° = 144°$

8. In the given figure, if l || n, find the value of x.



Sol. Draw a line 'n' through O such that n || l and n || m.

As l || n, OP is transversal.



$\Rightarrow$ $\angle 1 = 35°$ (Alternate interior angles)

Also, n || m, OQ is transversal

$\angle 2 = 40°$ (Alternate interior angles)

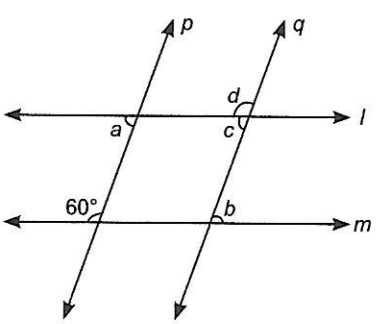
$\therefore$ $\angle POQ = \angle 1 + \angle 2 = 35° + 40° = 75°$

So, x = reflex $\angle POQ$

$= 360° - \angle POQ = 360° - 75° = 285°$

Short Answer Type Questions II [3 Marks]

9. Lines l || m and p || q in the given figure, then find the value of a, b, c and d.



Sol. Given l || m and p is transversal

$\Rightarrow$ $a + 60° = 180°$

(Co-interior angles on the same side of transversal)

$\Rightarrow$ $a = 120°$

But p || q (Given)

$\Rightarrow$ $c = a = 120°$ (Corresponding angles)

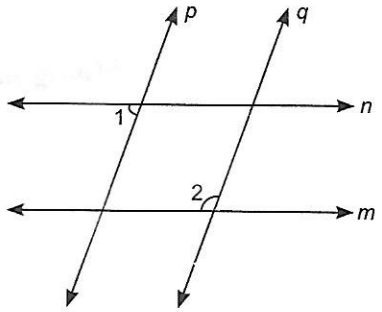
and $c = b$ (Alternate interior angles as l || m)

$\Rightarrow$ $b = 120°$

Also, $c + d = 180°$ (Linear pair axiom)

$\Rightarrow$ $d = 180° - c = 180° - 120° = 60°$

10. In the given figure, $n \parallel m$ and $p \parallel q$ of $\angle 1 = 75°$, prove that $\angle 2 = \angle 1 + \frac{1}{3}$ of a right angle.



Sol. Given: $\angle 1 = 75°$

Now, $m \parallel n$ and p is transversal

$\Rightarrow \quad \angle 1 + \angle 3 = 180°$ (Co-interior angles)

$\Rightarrow \quad 75° + \angle 3 = 180°$

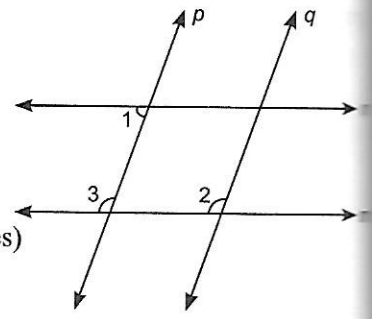
$\Rightarrow \quad \angle 3 = 180° - 75° = 105°$

Now, $p \parallel q$ and m is transversal

$\Rightarrow \quad \angle 2 = \angle 3 = 105$ (Corresponding angles)

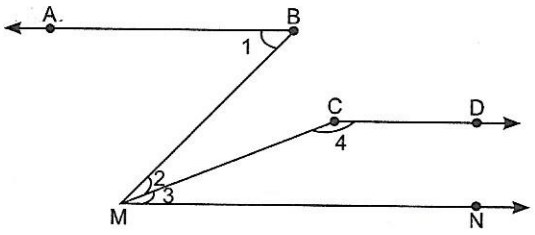
$= 75° + 30° = 75° + \frac{1}{3} \times 90°$

$\angle 2 = \angle 1 + \frac{1}{3} \times$ right angle.



Hence Proved

11. In the given figure, $\angle 1 = 55°$, $\angle 2 = 20°$, $\angle 3 = 35°$ and $\angle 4 = 145°$. Prove that AB $\parallel$ CD.



Sol. We have,

$\angle BMN = \angle 2 + \angle 3 = 20° + 35° = 55° = \angle 1 = \angle ABM.$

But these are the alternate angles formed by transversal BM on AB and MN.

So, by converse of alternate interior angles theorem.

AB $\parallel$ MN ...(*i*)

Now, $\angle 3 + \angle 4 = 35° + 145° = 180°$

This, shows that sum of the co-interior angles is 180°.

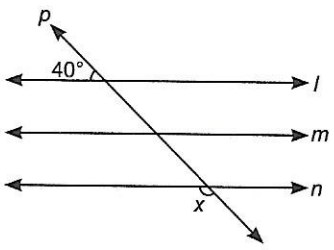
Hence, CD $\parallel$ MN ...(*ii*)

From (*i*) and (*ii*), we have AB $\parallel$ CD.

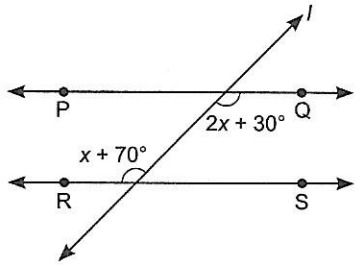
Hence proved

PRACTICE QUESTIONS BASED ON EXERCISE 6.2

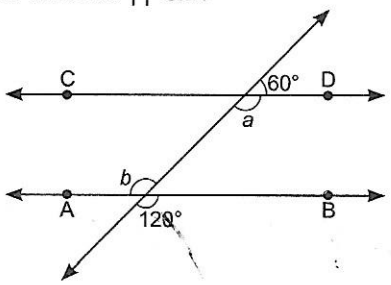
1. State the conditions when a transversal intersects two parallel lines.
2. In the given figure, $l \parallel m \parallel n$ and p is transversal. Find the value of x.



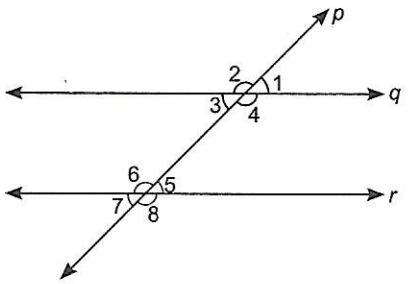
3. In the figure PQ || RS, and l is transversal, then what will be the value of x?



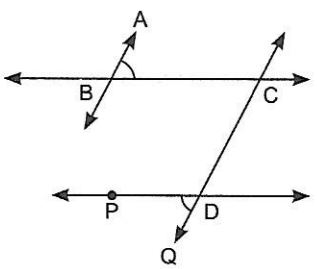
4. In the given figure, find the value of a and b and then show that AB || CD.



5. In the given figure, p is transversal to q and r. Given $q \parallel r$ and $\angle 1 = 75°$. Find $\angle 6$ and $\angle 7$.

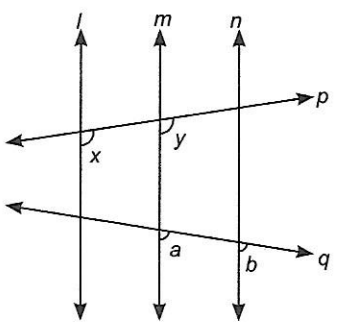


6. In the given figure, AB || CQ and BC || PD. Prove that $\angle ABC = \angle QDP$.

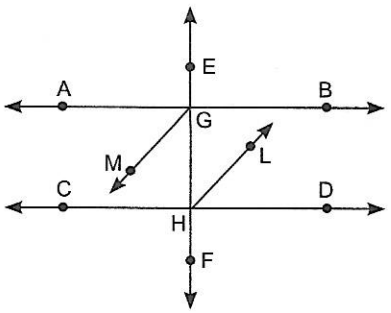


7. Prove that two lines perpendicular to the same line are parallel to each other.

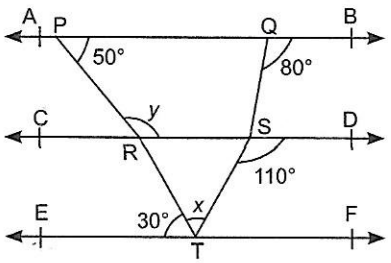
8. In the given figure, if $a = b$ and $x = y$ prove that $l \parallel n$.



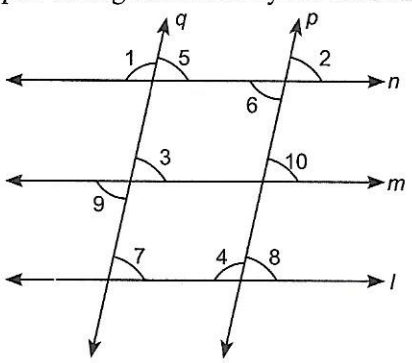
9. In the given figure, bisector GM and HL of alternate angles AGH and DHG respectively are parallel to each other. Prove that AB || CD.



10. In the given figure, AB || CD || EF. Find the value of $(y + x) : (y - x)$. **[CBSE 2014]**

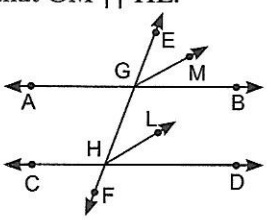


11. In the given figure, $l \parallel m \parallel n$ and $p \parallel q$. Name the type of pair of angles formed by the following:



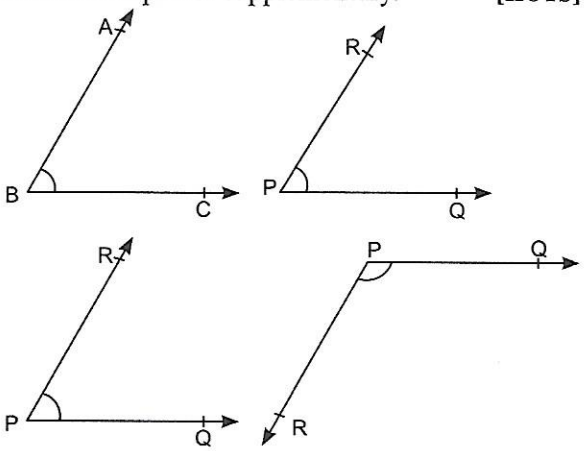
(a) $\angle 1, \angle 5$,
(b) $\angle 2, \angle 8$
(c) $\angle 2, \angle 6$
(d) $\angle 4, \angle 7$
(e) $\angle 9, \angle 3$
(f) $\angle 9, \angle 7$
(g) $\angle 6, \angle 10$
(h) $\angle 5, \angle 7$

12. In the given figure, EF is the transversal to two parallel lines AB and CD. GM and HL are the bisectors of the corresponding angles EGB and EHD. Prove that GM || HL.

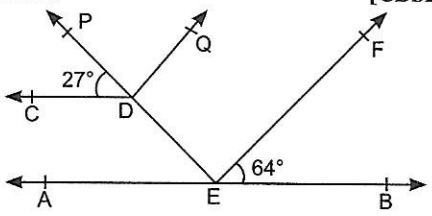


13. Prove that two lines that are respectively parallel to two intersecting lines intersect each other. **[HOTS]**

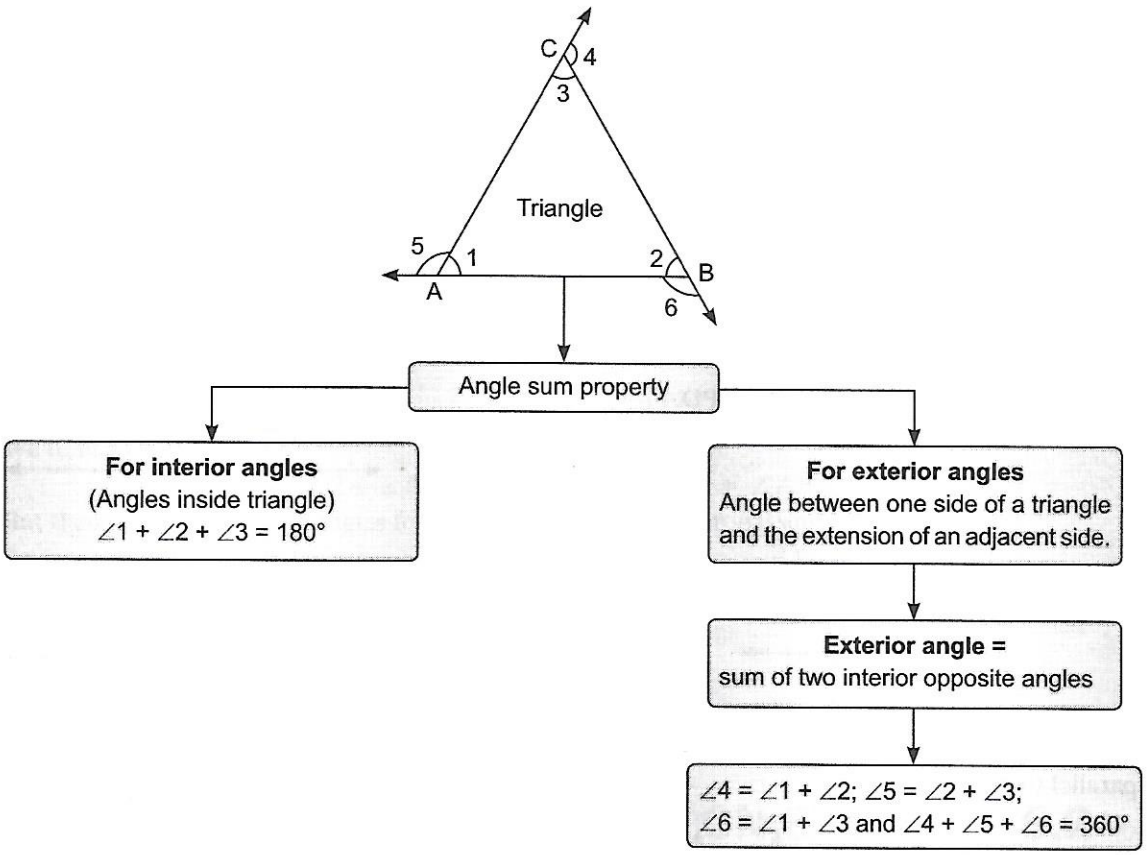
14. Prove that if the arms of one angle are respectively parallel to the arms of another angle, then the angles are either equal or supplementary. **[HOTS]**



15. In the given figure, EF || DQ and AB || CD. If ∠FEB = 64°, ∠PDC = 27°, then find ∠PDQ, ∠AED and ∠DEF. **[CBSE 2010]**



Angle Sum Property of Triangle and Exterior Angle Property

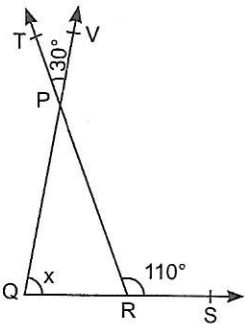


- The sum of the angles of a triangle is 180°.
- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.
- Exterior angle of a triangle is greater than either of its interior opposite angles.

SOLVED QUESTIONS BASED ON EXERCISE 6.3

Very Short Answer Type Questions [1 Mark]

1. In the given figure, find the $\angle x$.



Sol. We have $\angle QPR = \angle TPV$ ...(Vertically opposite angles)

$\Rightarrow \angle QPR = 30°$

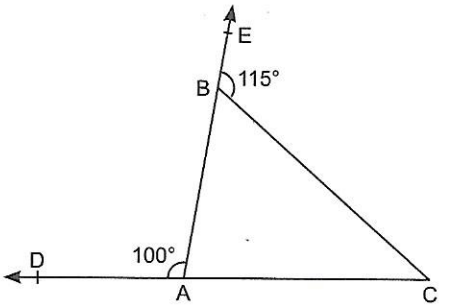
From exterior angle theorem,

$\angle QPR + \angle PQR = \angle PRS$

$\Rightarrow 30° + x = 110°$

$\Rightarrow x = 110° - 30° = 80°$

2. In the given figure, $\angle EBC = 115°$ and $\angle DAB = 100°$. Find $\angle ACB$. **[CBSE 2014]**



Sol. From the figure, $\angle EBC + \angle ABC = 180°$ (Linear pair axiom)

$\Rightarrow 115° + \angle ABC = 180°$

$\Rightarrow \angle ABC = 180° - 115° = 65°$

Now, in ΔABC, $\angle BAD = \angle ABC + \angle ACB$

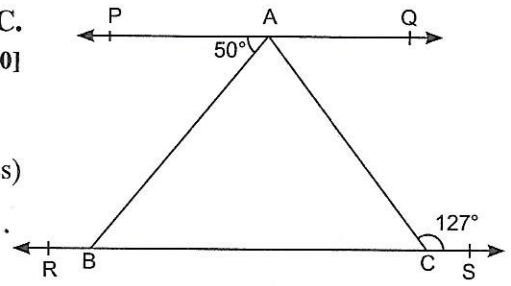
(Exterior angle theorem)

$\Rightarrow 100° = 65° + \angle ACB$

$\Rightarrow \angle ACB = 100° - 65° = 35°$

Hence, $\angle ACB = 35°$.

3. In the given figure, PQ || RS and $\angle ACS = 127°$, find $\angle BAC$. **[CBSE 2010]**



Sol. Given: PQ || RS

AC is transversal

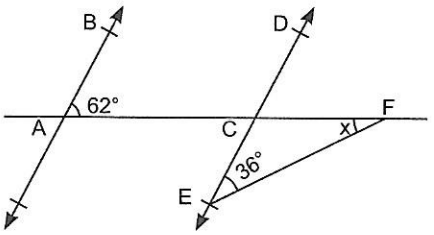
$\Rightarrow \angle PAC = \angle ACS$ (Alternate interior angles)

$\Rightarrow \angle PAB + \angle BAC = \angle ACS$

$\Rightarrow 50° + \angle BAC = 127°$

$\Rightarrow \angle BAC = 127° - 50° = 77°$.

4. In the given figure AB || ED, find x. **[CBSE 2010]**



Sol. Given: AB || ED

$\therefore \angle DCF = \angle BAC = 62°$ (Corresponding angles)

But $\angle DCF = \angle CEF + \angle CFE$ (Exterior angle theorem)

$\Rightarrow 62° = 36° + x$

$\Rightarrow x = 62° - 36° = 26°$.

5. The angle of a triangle ABC are in the ratio 2 : 3 : 4. Find the largest angle of the triangle. **[CBSE 2016]**

Sol. Given: $\angle A : \angle B : \angle C = 2 : 3 : 4$

Let $\angle A = 2x, \angle B = 3x, \angle C = 4x$

Using angle sum property of a triangle, we have

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 2x + 3x + 4x = 180°$

$\Rightarrow 9x = 180°$

$\Rightarrow$ $x = 20°$

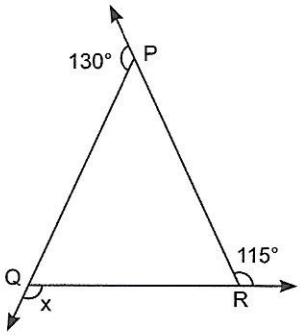
$\therefore$ $\angle A = 2x = 2 \times 20° = 40°$

$\angle B = 3x = 3 \times 20° = 60°$

$\angle C = 4x = 4 \times 20° = 80°$

Hence, largest angle of the triangle is 80°.

6. In the given figure, find the value of x.



Sol. We know that sum of exterior angle of ΔPQR is 360°

$\Rightarrow$ $130° + x + 115° = 360°$

$\Rightarrow$ $245° + x = 360°$

$\Rightarrow$ $x = 360° - 245° = 115°$

Short Answer Type Questions I [2 Marks]

7. Two angles of triangle are equal and the third angle is greater than each of these angles by 30°. Find all the angles of the triangle.

Sol. Let each of the two equal angles be x. According to the question, third angle $= x + 30°$.

Now, sum of angles of $\Delta = 180°$ (Angle sum property of a triangle)

$\Rightarrow$ $x + x + x + 30° = 180°$

$\Rightarrow$ $3x = 180° - 30° = 150°$

$\Rightarrow$ $x = \frac{150°}{3} = 50°$

Thus, angles of triangle are 50°, 50° and 80° respectively.

8. One of the angles of triangle is 75°, find the remaining two angles if their difference is 35°.

Sol. Let in ΔABC, $\angle A = 75°$ and $\angle B - \angle C = 35° \Rightarrow \angle B = \angle C + 35°$

Now, $\angle A + \angle B + \angle C = 180°$ (Angle sum property of a triangle)

$\Rightarrow$ $75° + \angle C + 35° + \angle C = 180°$

$\Rightarrow$ $110° + 2\angle C = 180°$

$\Rightarrow$ $2\angle C = 180° - 110° = 70°$

$\Rightarrow$ $\angle C = \frac{70°}{2} = 35°$ and $\angle B = \angle C + 35° = 35° + 35° = 70°$

9. Prove that if one angle of a triangle is equal to the sum of the other two angles, then the triangle is right angled.

Sol. Given: In ΔABC, $\angle A = \angle B + \angle C$

Now, $\angle A + \angle B + \angle C = 180°$ (Angle sum property of a triangle)

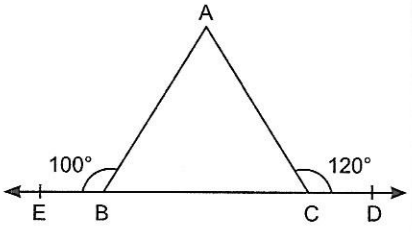
$\Rightarrow$ $\angle A + (\angle B + \angle C) = 180°$

$\Rightarrow$ $\angle A + \angle A = 180°$

$\Rightarrow$ $2\angle A = 180° \Rightarrow \angle A = \frac{180°}{2} = 90°$

Hence with $\angle A = 90°$, the given triangle is right angled triangle.

10. The exterior angles obtained on producing the base of a triangle both ways are 100° and 120°. Find all the angles. **[CBSE 2011]**



Sol. In ΔABC, $\angle ABE + \angle ABC = 180°$ (Linear pair axiom)

$\Rightarrow$ $100° + \angle ABC = 180°$

$\Rightarrow$ $\angle ABC = 180° - 100° = 80°$

Similarly, $\angle ACB + \angle ACD = 180°$ (Linear pair axiom)

$\Rightarrow$ $\angle ACB + 120° = 180°$

$\Rightarrow$ $\angle ACB = 180° - 120° = 60°$

Now, again in ΔABC,

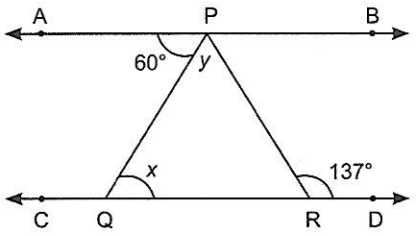
$\angle ABC + \angle ACB + \angle BAC = 180°$ (Angle sum property of triangle)

$\Rightarrow$ $80° + 60° + \angle BAC = 180°$

$\Rightarrow$ $\angle BAC = 180° - 140° = 40°$

Hence, $\angle BAC = 40°$, $\angle ABC = 80°$ and $\angle ACB = 60°$

11. In the given figure, if AB || CD, $\angle APQ = 60°$ and $\angle PRD = 137°$, then find the value of x and y. **[CBSE 2010]**



Sol. Given AB || CD

PQ is transversal

$\Rightarrow$ $\angle APQ = \angle PQR$ (Alternate interior angles)

$\Rightarrow$ $60° = x$

Again in ΔPQR, exterior angle is $\angle PRD$

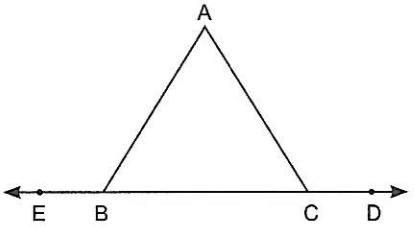
So, $\angle PRD = \angle PQR + \angle QPR$ ($\because$ Exterior angle theorem)

$\Rightarrow$ $137° = x + y$

$\Rightarrow$ $137° = 60° + y$

$\Rightarrow$ $y = 137° - 60° = 77°$

12. In the given figure, side BC of ΔABC is produced in both the directions. Prove that the sum of two exterior angles so formed is greater than 180°. **[CBSE 2013]**



Sol. The exterior angles in the given ΔABC are $\angle ABE$ and $\angle ACD$

To prove: $\angle ABE + \angle ACD > 180°$

Proof: In ΔABC,

$\angle 5 = \angle 1 + \angle 3$ ...(*i*) (Exterior angle theorem)

and $\angle 4 = \angle 1 + \angle 2$ ...(*ii*)

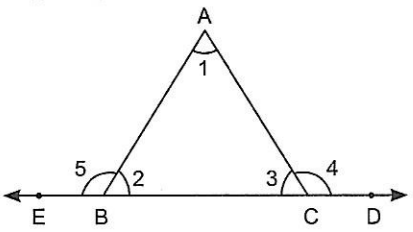
Adding (i) and (ii), we get

$\angle 4 + \angle 5 = \angle 1 + \angle 3 + \angle 1 + \angle 2$

$= \angle 1 + (\angle 1 + \angle 2 + \angle 3)$

$= \angle 1 + 180°$

$\Rightarrow$ $\angle 4 + \angle 5 > 180°$



(Angle sum property of a triangle)

Hence proved.

Short Answer Type Questions II [3 Marks]

13. In ΔABC, the bisector of $\angle B$ and $\angle C$ meets at O. Prove that $\angle BOC = 90 + \frac{\angle A}{2}$ **[CBSE 2014]**

Sol. Given the bisector of $\angle B$ and $\angle C$ of ΔABC meet at O as shown in figure.

OB is bisector of $\angle B$

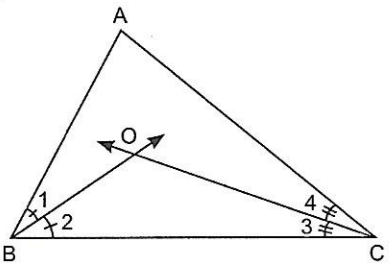
$\Rightarrow$ $\angle 1 = \angle 2 = \frac{1}{2}\angle ABC = \frac{1}{2}\angle B$

Similarly, OC is bisector of $\angle C$.

$\Rightarrow$ $\angle 3 = \angle 4 = \frac{1}{2}\angle ACB = \frac{1}{2}\angle C$

Now, in ΔABC,

$\angle A + \angle B + \angle C = 180°$



(Angle sum property of a triangle)

$\Rightarrow \quad \angle B + \angle C = 180° - \angle A$

$\Rightarrow \quad \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90° - \frac{\angle A}{2}$

$\Rightarrow \quad \angle 2 + \angle 3 = 90° - \frac{\angle A}{2}$

In ΔBOC,

$\angle OBC + \angle BOC + \angle BCO = 180°$ (Angle sum property of a triang

$\Rightarrow \quad \angle 2 + \angle BOC + \angle 3 = 180°$

$\Rightarrow \quad (\angle 2 + \angle 3) + \angle BOC = 180°$

$\Rightarrow \quad 90° - \frac{\angle A}{2} + \angle BOC = 180°$

$\Rightarrow \quad \angle BOC = 180° - 90° + \frac{\angle A}{2}$

$\Rightarrow \quad \angle BOC = 90° + \frac{\angle A}{2}$ Hence prov

14. The sides EF, FD and DE of a triangle DEF are produced in order forming three exterior angles DF EDQ and FER respectively. Prove that

$$\angle DFP + \angle EDQ + \angle FER = 360°$$

Sol. By using exterior angle theorem, we have

$\angle 4 = \angle 1 + \angle 2$ ...(*i*)

$\angle 5 = \angle 2 + \angle 3$ ...(*ii*)

and $\angle 6 = \angle 1 + \angle 3$ ...(*iii*)

Adding (*i*), (*ii*) and (*iii*), we get

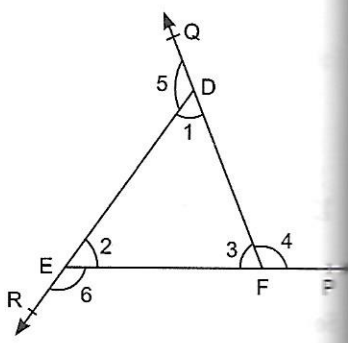
$\angle 4 + \angle 5 + \angle 6 = (\angle 1 + \angle 2) + (\angle 2 + \angle 3) + (\angle 1 + \angle 3)$

$= 2(\angle 1 + \angle 2 + \angle 3)$

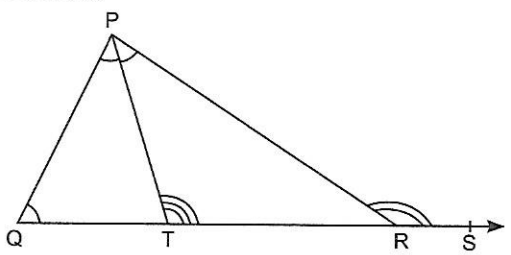
$= 2 \times 180°$ ($\because \angle 1 + \angle 2 + \angle 3 = 180°$

$= 360°$ Angle sum property of a triangle)

$\Rightarrow \angle DFP + \angle EDQ + \angle FER = 360°$ Hence proved.



15. Side QR of ΔPQR is produced to a point S as shown in the figure. The bisector of $\angle P$ meets QR at Prove that $\angle PQR + \angle PRS = 2\angle PTR$. [HOT



Sol. $\angle PRS$ is the exterior of ΔPQR

$\therefore \quad \angle PRS = \angle QPR + \angle PQR$ (Exterior angle theore

$= 2\angle TPQ + \angle PQR$

Adding $\angle PQR$ on both sides, we get [PT is bisector of $\angle P \therefore \angle TPQ = \frac{1}{2}\angle QP$

$\angle PQR + \angle PRS = \angle PQR + 2\angle TPQ + \angle PQR$

$= 2(\angle TPQ + \angle PQR)$

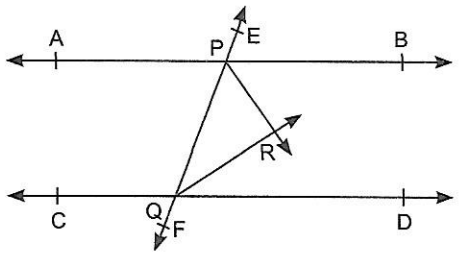
Now in ΔPTQ, $\angle PTR$ is exterior angle

$\therefore \quad \angle PTR = \angle TPQ + \angle PQR$

Thus from (i) and (ii), we get

$\angle PQR + \angle PRS = 2\angle PTR$ Hence prov

16. In the given figure, AB and CD are two parallel lines intersected by a transversal EF. Bisector of interior angles BPQ and DQP intersect at R. Prove that $\angle PRQ = 90°$



Sol. Given AB || CD and EF is transversal

$\therefore$ $\angle BPQ + \angle DQP = 180°$ (Interior angles on the same side of transversal is supplementary)

$\Rightarrow$ $\frac{1}{2}\angle BPQ + \frac{1}{2}\angle DQP = 180° \times \frac{1}{2} = 90°$ ...(*i*)

Now, PR is the bisector of $\angle BPQ$

$\Rightarrow$ $\angle RPQ = \frac{1}{2}\angle BPQ$

and QR is the bisector $\angle DQP$.

$\Rightarrow$ $\angle PQR = \frac{1}{2}\angle DQP$

From (*i*), we have $\angle RPQ + \angle PQR = 90°$ ...(*ii*)

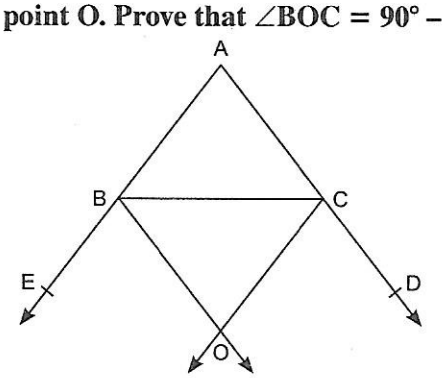
In ΔPQR, $\angle RPQ + \angle PQR + \angle PRQ = 180°$ (Angle sum property of a triangle)

$\Rightarrow$ $90° + \angle PRQ = 180°$

$\Rightarrow$ $\angle PRQ = 180° - 90° = 90°$ Hence proved.

Long Answer Type Questions [4 Marks]

17. In the given figure, bisectors of the exterior angles B and C formed by producing sides AB and AC of ΔABC intersect each other at the point O. Prove that $\angle BOC = 90° - \frac{1}{2}\angle A$.



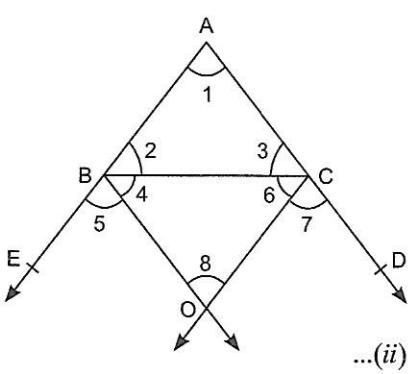
Sol. Ray BO is the bisector of $\angle CBE$

$\Rightarrow$ $\angle 4 = \angle 5 = \frac{1}{2}\angle CBE$

Now, $\angle 2 + \angle 4 + \angle 5 = 180°$ (Linear pair axiom)

$\Rightarrow$ $\angle 2 + 2\angle 4 = 180°$ ($\because \angle 4 = \angle 5$)

$\Rightarrow$ $\angle 4 = 90° - \frac{\angle 2}{2}$ ...(*i*)



Similarly, ray OC bisect $\angle BCD$

$\therefore$ $\angle 6 = \frac{1}{2}\angle BCD = \frac{1}{2}(180° - \angle 3) = 90° - \frac{\angle 3}{2}$ ...(*ii*)

Now, in ΔBOC,

$\angle 4 + \angle 6 + \angle 8 = 180°$ (Angle sum property of a triangle)

$\Rightarrow \quad \left(90° - \frac{\angle 2}{2}\right) + \left(90° - \frac{\angle 3}{2}\right) + \angle 8 = 180°$

$\Rightarrow \quad \angle 8 = \frac{1}{2}(\angle 2 + \angle 3)$

Again in ΔABC,

$\angle 1 + \angle 2 + \angle 3 = 180°$ (Angle sum property of a tria

$\Rightarrow \quad \angle 2 + \angle 3 = 180° - \angle 1$

Substituting in (iii), we get $\quad \angle 8 = \frac{1}{2}(180 - \angle 1)$

$\Rightarrow \quad \angle 8 = 90° - \frac{\angle 1}{2}$

or $\quad \angle BOC = 90° - \frac{\angle BAC}{2}$ or $\angle BOC = 90° - \frac{1}{2}\angle A$ Hence pro

18. Side BC, CA and BA of triangle ABC produced to D, Q, P respectively as shown in the figure. If $\angle ACD = 100°$ and $\angle QAP = 35°$, find all the angles of a triangle. [CBSE 20

Sol. We have $\angle BAC = \angle QAP$

...(Vertically opposite angles)

$\Rightarrow \quad \angle BAC = 35°$

...(Given, that $\angle QAP = 35°$)

Also, $\angle ACB + \angle ACD = 180°$ (Linear pair axiom)

$\Rightarrow \quad \angle ACB + 100° = 180°$

$\Rightarrow \quad \angle ACB = 180° - 100° = 80°$

In ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180°$

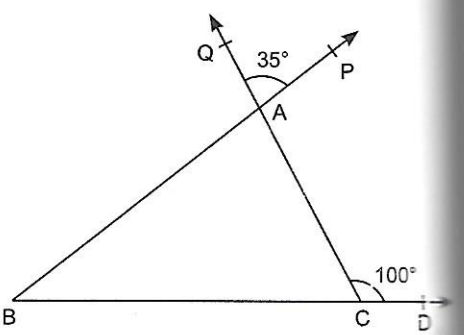
(Angle sum property of a triangle)

$\angle ABC + 80° + 35° = 180°$

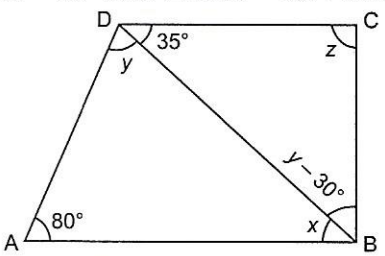
$\angle ABC + 115° = 180°$

$\Rightarrow \quad \angle ABC = 180° - 115° = 65°$

Hence, $\angle ABC = 65°$, $\angle BAC = 35°$ and $\angle ACB = 80°$



19. In the given figure, AB || DC, $\angle BDC = 35°$ and $\angle BAD = 80°$. Find x, y, z.



Sol. Given: AB || DC

BD is transversal

$\Rightarrow \quad x = 35°$ (Alternate interior angles

In ΔABD, $\angle ABD + \angle ADB + \angle BAD = 180°$ (Angle sum property of a triangle

$\Rightarrow \quad x + y + 80° = 180°$

$\Rightarrow \quad 35° + y + 80° = 180°$ ($\because x = 35°$

$\Rightarrow \quad y = 180° - 115° = 65°$

$\therefore \quad \angle DBC = y - 30° = 65° - 30° = 35°$

Again in ΔBCD,

$\angle DBC + \angle BCD + \angle CDB = 180°$ (Angle sum property of a triangle

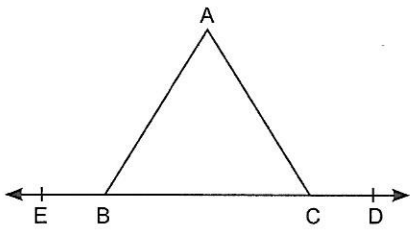
$\Rightarrow \quad 35° + z + 35° = 180°$

$\Rightarrow \quad z = 180° - 70° = 110°$

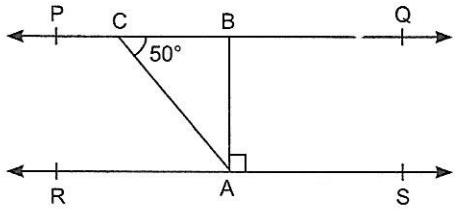
Hence, $x = 35°, y = 65°$ and $z = 110°$.

PRACTICE QUESTIONS BASED ON EXERCISE 6.3

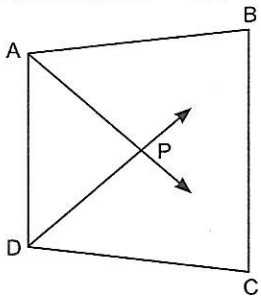
1. An exterior angle of a triangle is 115° and one of the interior opposite angles is 60°. Find the other interior opposite angle.
2. In equilateral triangle PQR, if QR is extended to S, find the value of exterior angle ∠PRS.
3. In the given figure, if ∠ABC = ∠ACB, then which angle would be equal to ∠ABE?



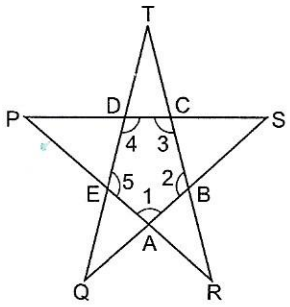
4. The angles of triangle are in the ratio 3 : 4 : 5. Find the measure of each angle.
5. In the given figure PQ || RS, BA ⊥ RS and ∠BCA = 50° find ∠BAC and ∠CAS.



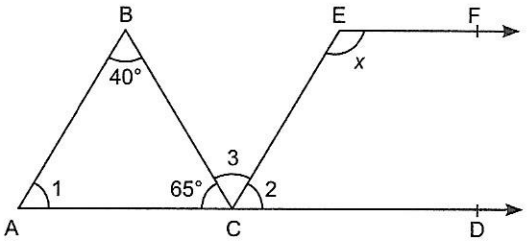
6. In the given figure, AP and DP are the bisectors of two adjacent angles A and D of a quadrilateral ABCD. Prove that 2∠APD = ∠B + ∠C. **[HOTS]**



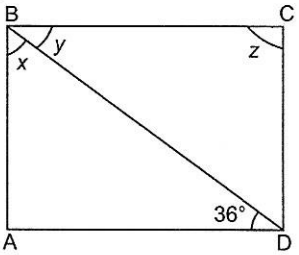
7. In ΔABC, ∠B= 45°, ∠C = 55° and bisector of ∠A meet BC at point D. Find ∠ADB and ∠ADC. **[HOTS]**
8. In the given figure, prove that ∠P + ∠Q + ∠R + ∠S + ∠T = 2 right angles.



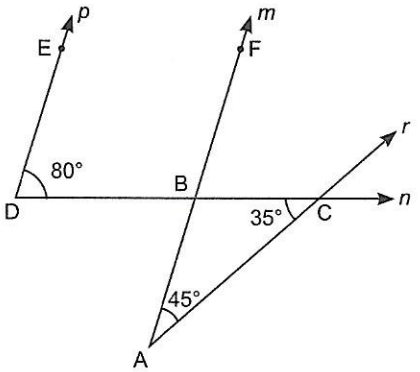
9. In the figure, AB || CE, CD || EF. Find the value of x.



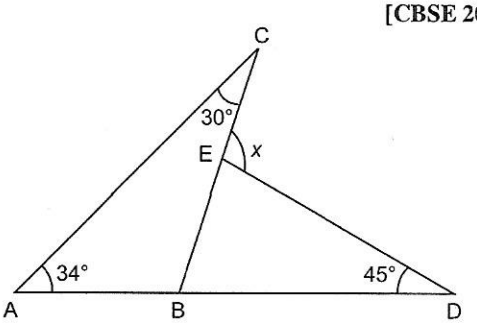
10. In the given figure, AB || DC, $x = \frac{4y}{3}$ and $y = \frac{3z}{8}$. Find ∠BCD, ∠ABC and ∠BAD.



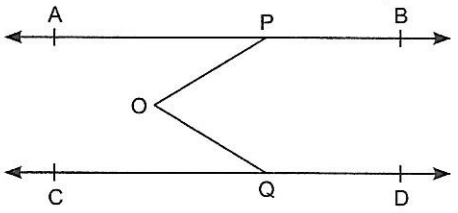
11. In the given figure, prove that p || m.



12. In the given figure, find the value of x. **[CBSE 2016]**

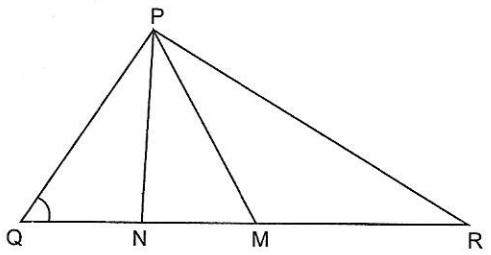


13. In the given figure, AB || CD. Prove that ∠BPO + ∠POQ + ∠DQO = 360°. **[CBSE 2016]**



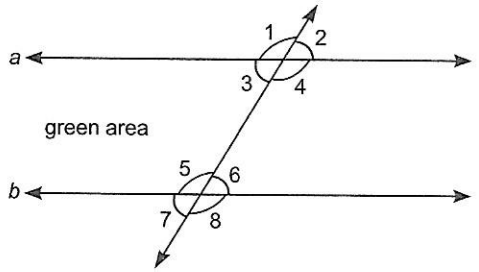
14. Prove that the sum of three angles of a triangle is two right angles. If in a right angled triangle, an acute angle is one fourth of the other. Find the acute angle.

15. In the given figure, $\angle Q > \angle R$ and M is a point on QR such that PM is the bisector of $\angle QPR$. If the perpendicular from P on QR meets at N, then prove that $\angle MPN = \frac{1}{2}(\angle Q - \angle R)$. **[HOTS]**



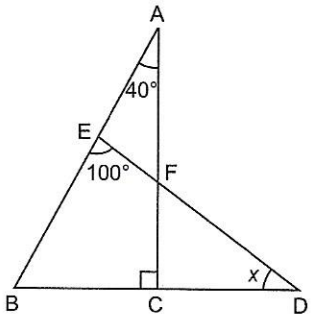
Value Based Questions

1. Builder has made a layout of a colony so that lane '*a*' is parallel to lane '*b*'? In between lanes '*a*' and '*b*', he plans to leave green area as shown in figure.
   (*i*) What values is he showing by doing so?
   (*ii*) If measure of $\angle 1$ is 120°. Find the measure of all other angles. **[CBSE 2016]**



2. A medicinal plant garden owner decided to construct a track along the side of the triangular area and three exit tracks. His friend suggested him that the ratio of an exterior angle at which the exit track formed must be in 21 : 25 : 26 for the beauty of region and easiness of pedestrian.
   (*i*) Find the values of exterior angles at which exit track would be constructed.
   (*ii*) What is the importance of green plants?
   (*iii*) What values are depicted by the owner?

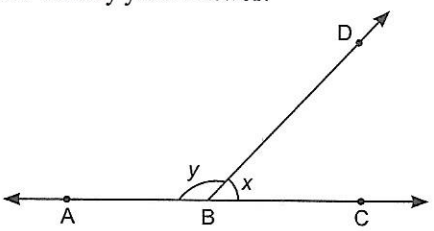
3. A villager Ram has a plot of land in the shape of concave quadrilateral ABDFA as shown in the figure. He himself decided to construct a health care centre for the villagers in the area FCD and keep the remaining with him.
   (*i*) Find the value of *x*.
   (*ii*) What values of Ram is depicting while deciding to do so?



INTEGRATED EXERCISE

Very Short Answer Type Questions [1 Mark]

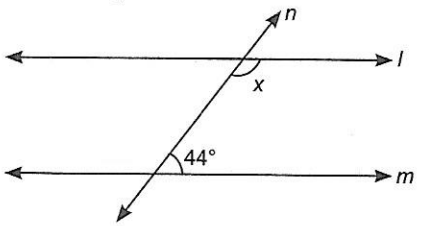
1. For what value of $x + y$ in the given figure, will ABC be a line? Justify your answer.



2. Two lines l and m are perpendicular to the same line n. Are l and m perpendicular to each other? Give reason for your answer.

3. How many triangles can be drawn having its angles as 45°, 64° and 72°?

4. In the given figure, find the value of x, for which the lines l and m are parallel.

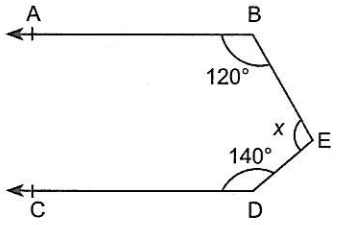


5. A transversal l intersects two lines p and q such that a pair of alternate exterior angles are equal. Then what can you say about the lines p and q?

6. A transversal l intersects two lines m and n such that the sum of angles on the same side of it is supplementary. Then, what can you say about the lines m to n?

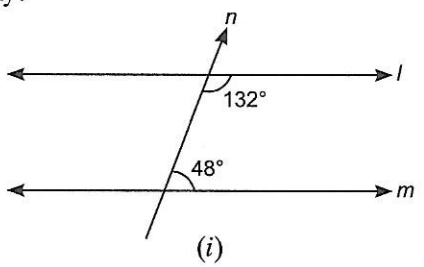
7. In the given figure, AB||CD. Find the value of x.

[CBSE 2013]

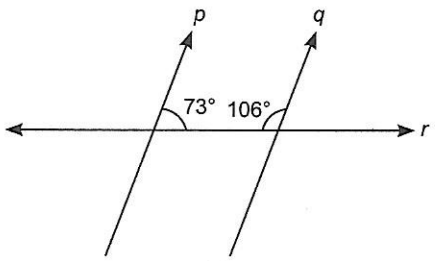


Short Answer Type Questions I [2 Marks]

8. In the given figure, which of the two lines are parallel and why?



(*i*)



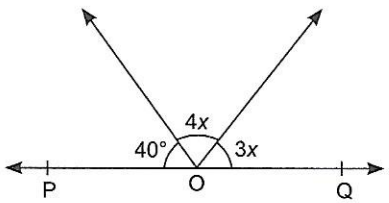
(*ii*)

9. If one angle of a triangle is 130°, then what is the angle between the bisectors of the other two angles?

[NCERT Exemplar]

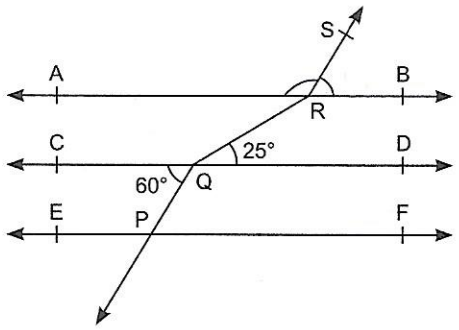
10. In the given figure, POQ is a line. Find the value of x.

[NCERT Exemplar]



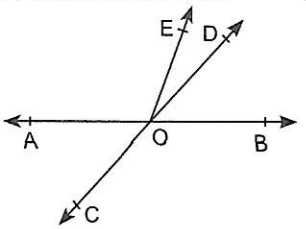
11. If the angles of triangle are in the ratio 5 : 3 : 7, then what will be the nature of triangle?

12. In the given figure, if AB || CD || EF, PQ || RS, $\angle RQD = 25°$ and $\angle CQP = 60°$. Find $\angle QRS$



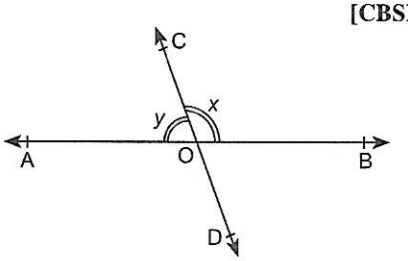
13. In the given figure, two lines AB and CD intersect at O. OE is any ray such that $\angle BOE = 80°$. If $\angle BOD = 30°$, then find $\angle EOC$.

[CBSE 2014]



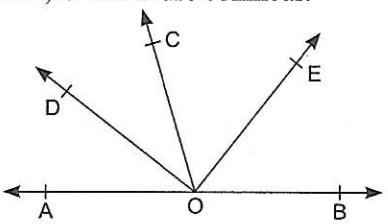
14. In the figure below, angles x and y are supplementary angles. If $\angle x = 110°$, find the value of y.

[CBSE 2011]

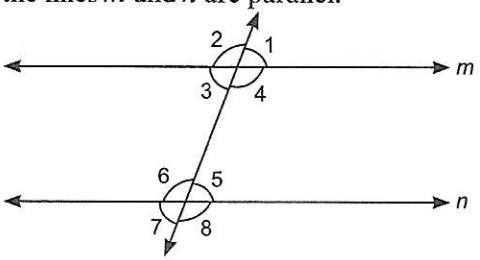


Short Answer Type Questions II [3 Marks]

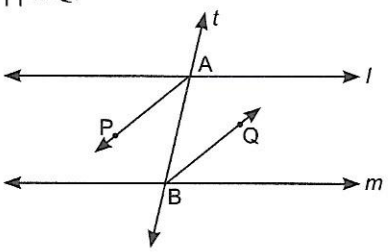
15. In the given figure, OD is bisector of $\angle AOC$, OE is the bisector of $\angle BOC$ and OD $\perp$ OE. Show that the points A, O and B are collinear.



16. In the given figure, $\angle 1 = 60°$ and $\angle 6 = 120°$. Show that the lines m and n are parallel.

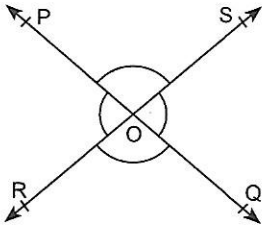


17. AP and BQ are the bisectors of two alternate interior angles formed by the intersection of a transversal t with parallel lines l and m (in the given figure) show that AP || BQ.



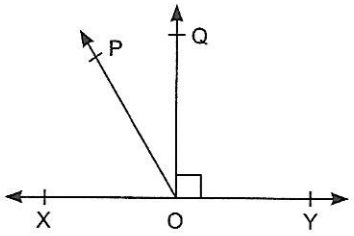
18. A ΔABC is right angled at A. L is point on BC such that AL $\perp$ BC. Prove that $\angle BAL = \angle ACB$.

19. In the given figure, line PQ and RS intersect each other at point O. If $\angle POR : \angle ROQ = 4 : 5$. Find $\angle POR$ and $\angle ROQ$.

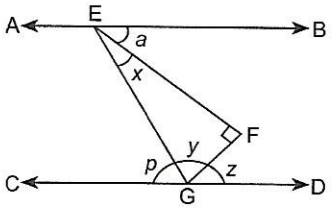


20. In given figure, XOY is a straight line and OQ $\perp$ XY at O. Show that $2\angle QOP = \angle YOP - \angle XOP$.

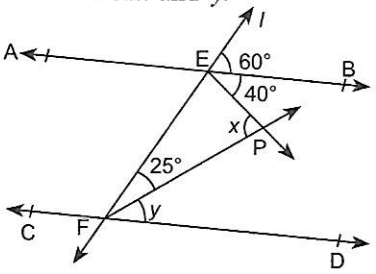
[CBSE 2010]



21. In the given figure $x : y = 3 : 2$ and $y + z = 100°$ and AB || CD, find $\angle a$.



22. In the adjoining figure, AB || CD and l is transversal. Find the value of x and y. **[CBSE 2016]**



Long Answer Type Questions [4 Marks]

23. If two lines intersect, prove that the vertically opposite angles are equal. **[CBSE 2011, 2014]**

24. Prove that through a given point, we can draw only one perpendicular to a given line. **[NCERT Exemplar]**

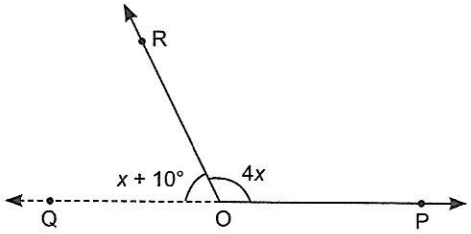
25. Prove that two lines that are respectively perpendicular to two intersecting lines intersect each other. **[NCERT Exemplar]**

26. Prove that a triangle must have atleast two acute angles.

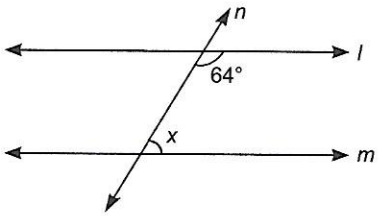
ASSESS YOURSELF

1. Can two distinct lines in a figure have more than one point in common?

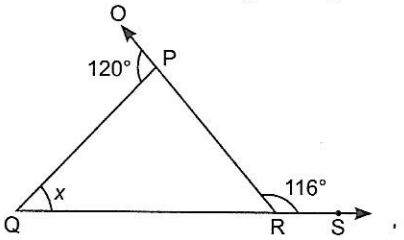
2. Given $\angle POR = 4x$ and $\angle QOR = x + 10°$. Find the value of x, for which POQ will be a straight line.



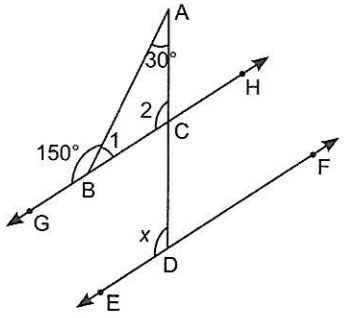
3. In the given figure, find the value of x for which the lines l and m are parallel.



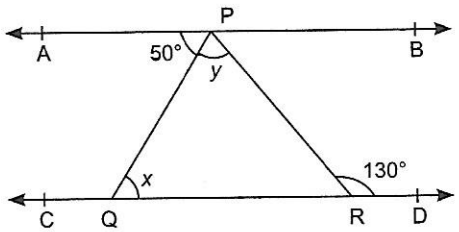
4. Compute the value of x in the given figure.



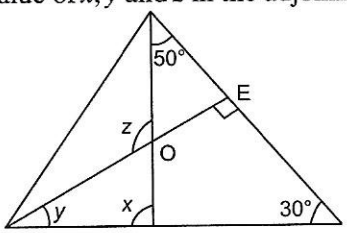
5. In the given figure, if BC || DE, $\angle ABG = 150°$ and $\angle BAD = 30°$, then compute the value of x.



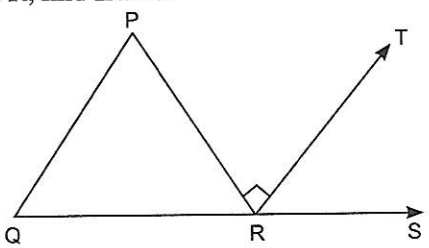
6. In the given figure, AB || CD, $\angle APQ = 50°$ and $\angle PRD = 130°$, compute the value of x and y.



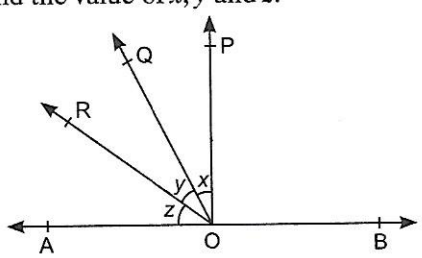
7. Find the value of x, y and z in the adjoining figure.



8. In the given figure, side QR of a ΔPQR has been produced to S. If $\angle P : \angle Q : \angle R = 3 : 2 : 1$ and $RT \perp PR$, find $\angle TRS$.

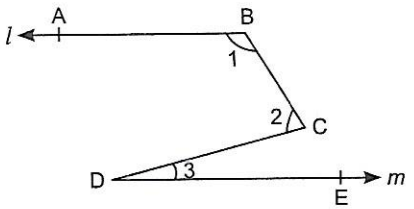


9. In the given figure, $PO \perp AB$. If $x : y : z = 1 : 3 : 5$, then find the value of x, y and z.

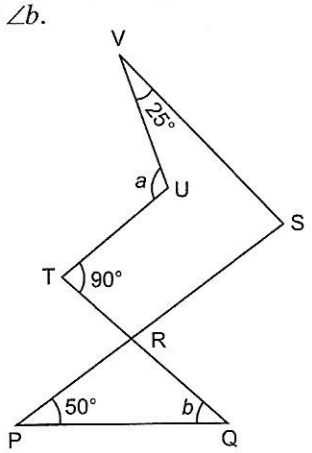


10. For spreading the message "Save Environment, Save Future", a rally was organized by some students of a school. They were given triangular carboard pieces ABC which they divide into two parts by drawing the angle bisectors BO and CO of base angle B and C. Prove that $\angle BOC = 90° + \frac{1}{2}\angle A$. What is the benefit of these types of rallies?

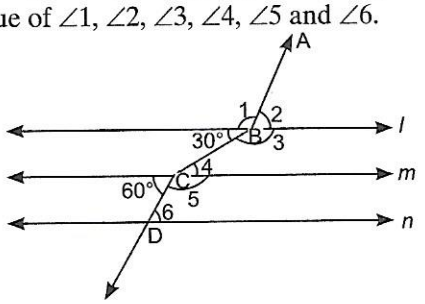
11. In the given figure, $l \,||\, m$. Show that $\angle 1 + \angle 2 - \angle 3 = 180°$. **[CBSE 2011]**



12. If in the given figure, TU || SR and TR || SV, then find $\angle a$ and $\angle b$. **[HOTS]**



13. In the given figure, $l \,||\, m \,||\, n$ and AB || CD. Find the value of $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$.



14. Prove that sum of three angles of a triangle is 180°. Using this result, find value of x and all the three angles of a triangle, if it is given that three angles of triangle are $(2x - 7)°$, $(x + 25)°$ and $(3x + 12)°$ respectively.

15. In the given figure, AB || CD and EF is a transversal, find x, y and z. **[CBSE 2011]**